§3. Pressure-Driven Global Marginal Eigenfunction in a Heliotron/Torsatron System

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As is well known, resonant ideal pressure-driven modes, namely interchange modes with the mode rational surfaces, have the δ-function like singular eigenfunction at the marginal stability. These characteristics come from the fact that the mode rational surfaces are regular singular points at the marginal stable state. Strictly speaking, it is true under the condition that the pressure gradient at the mode rational surface is finite. If the pressure gradient vanishes at the mode rational surface, then there is a possibility that the global marginal eigenfunction exists. The existence of the global marginal eigenfunction has been analytically shown. The linearized reduced MHD equation at the marginal stable state, namely the Newcomb equation (neglecting the toroidicity) is as follows:

$$(m\iota - n)\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{m^2}{r^2}\right\}\left[(m\iota - n)\tilde{u}\right]$$
$$-\frac{m^2}{r^2}\frac{\beta'\overline{\Omega}'}{2}\tilde{u} - \frac{m(m\iota - n)}{r}J'\tilde{u} = 0$$

where β, $\overline{\Omega}$, and J are equilibrium beta, the magnetic curvature, and net toroidal current, respectively:

$$\frac{1}{r}\frac{d}{dr}(r^2\iota_P) = J, \quad \iota = \iota_P + \iota_V,$$
$$\overline{\Omega}' = \frac{M}{L}\left[4r\iota_V + r^2\iota_V'\right] (\geq 0)$$

The subscripts indicate the vacuum quantities, L and M are the polarity and the toroidal field period of helical coils. Note that the average magnetic curvature stems only from the vacuum magnetic field, and that the rational transform ι consists of ι_P due to plasma net toroidal current J and ι_V due to the vacuum magnetic field. Let's consider the case with single mode rational surface. Since the Newcomb equation is solved independently in each interval separated by the mode rational surfaces, we seek the global eigenfunction, so that

$$\tilde{u}\begin{cases} \neq 0, & \text{for } 0 \leq r < r_s, \quad \iota < \iota_s \equiv \iota(r_s) = \dfrac{n}{m}, \\ = 0, & \text{for } r_s < r \leq 1, \quad \iota > \iota_s. \end{cases}$$

In order to obtain an analytical solution, let assume that in the inner region $(0 \leq r < r_s)$

$$\begin{aligned} \beta'(r) &= \beta_{in}'(r) &&= -C_\beta r(\iota_s - \iota)^2 \ (\leq 0) \\ J'(r) &= J_{in}'(r) &&= -C_J r(\iota_s - \iota) \ (\leq 0) \\ \overline{\Omega}'(r) &= \overline{\Omega}_{in}'(r) &&= \overline{\Omega}'' r, \overline{\Omega}'' = 4\frac{M}{L}\iota_V(0) \end{aligned}$$

where $C_\beta > 0$ and $C_J \geq 0$. Note that no special restriction is need in the outer region $(r_s < r \leq 1)$ except for continuity through the mode rational surface. Under these assumtions, we have a global marginal eigenfunction:

$$\tilde{u} = \begin{cases} \dfrac{J_m(\mathcal{E}r)}{\iota_s - \iota} & : \text{ for } 0 \leq r \leq r_s \\ 0 & : \text{ for } r_s < r \leq 1 \end{cases}$$

where

$$\begin{aligned} \mathcal{E} &\equiv \sqrt{\frac{\overline{\Omega}''}{2}C_\beta - C_J} = \frac{Z_{m,1}}{r_s}, \\ Z_{m,1} &: \text{ 1st zero point of } J_m(z) \end{aligned}$$

Above relation prescribes the parameter window of the global marginal function.

The Essential condition that the global marginal eigenfunctions exist is for the pressure gradient to vanish at the mode rational surface. This situation is easily established after saturation of high mode number ideal interchange modes with the same helicity and/or the fast resistive interchange modes not having the magnetic reconection. Moreover, even if magnetic islands exist at the mode rational surfaces, the pressure gradient also vanishes, so that the global marginal eigenfunctions are able to exist. From these consideration on the occurence condition, it may be concluded that the global marginal eigenfunctions similar to those of $m/n = 1/1$ modes in tokamaks universally exist in heliotron/torsatron systems.